Application of Birks' law of scintillator nonlinearity in Geant4

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Outline

• CALICE analogue hadronic calorimeter prototype
• Birks’ law of scintillator nonlinearity
• Measurement of Birks’ coefficient (MPIK Heidelberg)
• Birks’ law in Geant4
• Conclusion & Discussion
CALICE analogue HCAL

- Hadronic calorimeter prototype for next $e^+e^-$ collider
- Sampling calorimeter with small plastic (polystyrene) tiles as active material
- Monte Carlo studies ➔ Scintillator properties needed
- Nonlinearity: Birks’ law
- Currently Birks’ coefficient from ZEUS calorimeter scintillator is used
- Measure $kB$ for the AHCAL scintillator

![Plastic scintillator tile (3×3×0.5cm³)](image)
Birks’ Saturation Formula

- Specific energy loss $dE/dx$ is high before particle is stopped
- High ionization density $dI/dx \propto dE/dx$
- Quenching: Excited atoms can interact and may de-excite radiationless
- Light yield per unit length $dL/dx$ is reduced for high $dE/dx$
- Non-linearity described by Birks’ formula:

$$\frac{dL}{dx} = \frac{S \frac{dE}{dx}}{1 + kB \frac{dE}{dx}}$$

Electron collision stopping power (Berger-Seltzer eq.)
• PMT measures light yield

• Germanium detector measures Energy of Compton scattered photon $E_{Ge}$

  \[ E_{e^-} = 662 \text{ keV} - E_{Ge} \]

• Coincidence trigger PMT and Ge-detector

• Measured energy range of electrons
  \[ \sim 30 \text{ - } 140 \text{ keV} \]

• Thanks to Christoph Aberle and Stefan Wagner for the ability to use the setup

• Detailed setup description in [1]
• PMT measures light yield
• Germanium detector measures Energy of Compton scattered photon $E_{\text{Ge}}$
• Coincidence trigger PMT and Ge-detector
• Measured energy range of electrons ~ 30 - 140 keV

Thanks to Christoph Aberle and Stefan Wagner for the ability to use the setup
Detailed setup description in [1]
Total Light-Yield

We have measured total Light-Yield (LY)

**Problem:** $\frac{dL}{dx}$ not easily measurable

\[
\frac{dL}{dx} = \frac{S \frac{dE}{dx}}{1 + kB \frac{dE}{dx}}
\]

divide by $\frac{dE}{dx}$

\[
\frac{dL}{dE} = \frac{S}{1 + kB \frac{dE}{dx}}
\]

Light-Yield of a particle with energy $E_0$:

\[
LY(E_0) = S \int_{0}^{E_0} \frac{1}{1 + kB \frac{dE}{dx}} dE
\]
kB Measurement (standalone)

- **Experimental data:**
  Light-yield as function of electron energy

- **Fit with calculated Light yield**
  → Fit parameter $kB$

**Fit function:**

$$LY(E_0) = S \int_0^{E_0} \frac{1}{1 + kB \frac{dE}{dx}} dE$$

**Fit result:**

$kB = 0.0151$ cm/MeV

Currently used value:

$kB = 0.007943$ cm/MeV

M. Hirschberg et. al.,
Birks’ coefficient in Geant4

• Geant4 is step based:

\[ E_0 \quad E_1 \quad \ldots \quad E_{n-1} \quad E_n = 0 \]

\[ \begin{array}{cccc}
\text{dx}_1 & \text{dx}_2 & \ldots & \text{dx}_n \\
\end{array} \]

• Steps can’t be arbitrarily small because of computational effort

• Light-Yield in Geant4 (secondaries not considered):

\[ LY = S \sum_{i=1}^{n} \frac{1}{1 + kB \frac{(E_{i-1} - E_i)}{dx_i}} \]

• Converges only against integral for small steps.
  This is not the case for default settings in Geant4!
Birks’ coefficient in Geant4

- Geant4 is step based:

\[ E_0 \quad E_1 \quad \ldots \ldots \ldots \ldots \quad E_{n-1} \quad E_n = 0 \]

- Steps can’t be arbitrarily small because of computational effort

- Light-Yield in Geant4 (secondaries not considered):

\[ LY = S \sum_{i=1}^{n} \frac{1}{1 + kB \frac{(E_{i-1} - E_i)}{dx_i}} \neq S \int_{0}^{E_0} \frac{1}{1 + kB \frac{dE}{dx}} \, dE \]

- Converges only against integral for small steps. This is not the case for default settings in Geant4!
Birks’ coefficient in Geant4

\[
\frac{dL}{S} = \frac{dE_{vis}}{dE} = \frac{1}{1 + kB \frac{dE}{dx}}
\]

Curve shows 120keV e\(^-\) with \(kB=0.0151\text{cm/MeV}\)

Area under curve represents the visible energy
Birks’ coefficient in Geant4

\[ \frac{dL/S}{dE} = \frac{dE_{vis}}{dE} = \frac{1}{1 + kB \frac{dE}{dx}} \]

Curve shows 120keV e\(^-\) with 
\(kB=0.0151\text{cm/MeV}\)

Geant4 with default settings and 
\(kB=0.0151\text{cm/MeV}\)
Visible energy is overestimated!
Birks’ coefficient in Geant4

\[
\frac{dL/S}{dE} = \frac{dE_{vis}}{dE} = \frac{1}{1 + kB \frac{dE}{dx}}
\]

Curve shows 120keV e\(^-\) with \(kB=0.0151\text{cm/MeV}\)

Geant4 with default settings and \(kB=0.0151\text{cm/MeV}\)
Visible energy is overestimated!

Possible solution:
Use “effective” Birks’ coefficient to get the correct result
\(kB=0.0186\text{cm/MeV}\)
Birks’ coefficient in Geant4

default Geant4 settings:
\( \alpha = 0.2 \)

Final range = 1mm

qualitatively:

smaller steps \( \leq \) larger steps
Birks’ coefficient in Geant4

Main disadvantage: need different effective Birks’ coefficient for different parameter configuration.

default Geant4 settings:
\[ \alpha = 0.2 \]
Final range = 1mm

qualitatively:
smaller steps \[ \rightarrow \]
larger steps
Method Improvement

- Step-wise integral calculation of visible energy
- Use method of Monte Carlo integration (VEGAS algorithm)
- Use pre-calculated $dE/dX$ to increase speed

\[ LY = S \sum_{i=1}^{n} \frac{1}{1 + k_B \frac{(E_{i-1} - E_i)}{dx_i}} \]

\[ LY = S \sum_{i=1}^{n} \int_{E_i}^{E_{i-1}} \frac{1}{1 + k_B \frac{dE}{dx}} dE \]
Method Comparison

- Birks’ coefficient independent of final range parameter (step-size)

- Residual fluctuation from production of secondaries (delta-electrons)

**qualitatively:**
smaller steps ← larger steps
Conclusion & Outlook

• Measured Birks’ coefficient higher than currently used one
  Measured value: $kB = 0.0151 \text{cm/MeV}$ (“old”: $kB = 0.007943 \text{cm/MeV}$)

• Default Geant4: Effective Birks’ coefficient needed in order to describe data correctly ($kB$ is step-size dependent)

• Integral method allows to calculate precisely the visible energy deposition for each step ($kB$ is step-size independent)

• To Do

• Study influence of new $kB$ and new method on simulated particle showers

• Test performance of integral method
Reference(s)

Backup
Step-limitation in GEANT4

• Every registered process sets a limit on the step-size
• The smallest limit is chosen for each step
• For ionization process, **Stepping function** determines the maximum step-size allowed $\Delta x_{max}$

$$\Delta x_{max} = \begin{cases} 
\alpha R + f(\rho, \alpha), & R > \rho \quad \text{(high energy)} \\
R, & R < \rho \quad \text{(low energy)}
\end{cases}$$

- $\rho$: Final range  *(Default value: 1mm)*
- $\alpha$: $dR/R$  *(Default value: 0.2)*

$\Rightarrow$ Small values of $\rho$ and $\alpha$ result in a small step size
Step Function

Final range
\[ \rho_R = 1\text{mm} \]

"slope"
\[ \alpha_R = 0.2 \]