High temperature symmetry non-restoration

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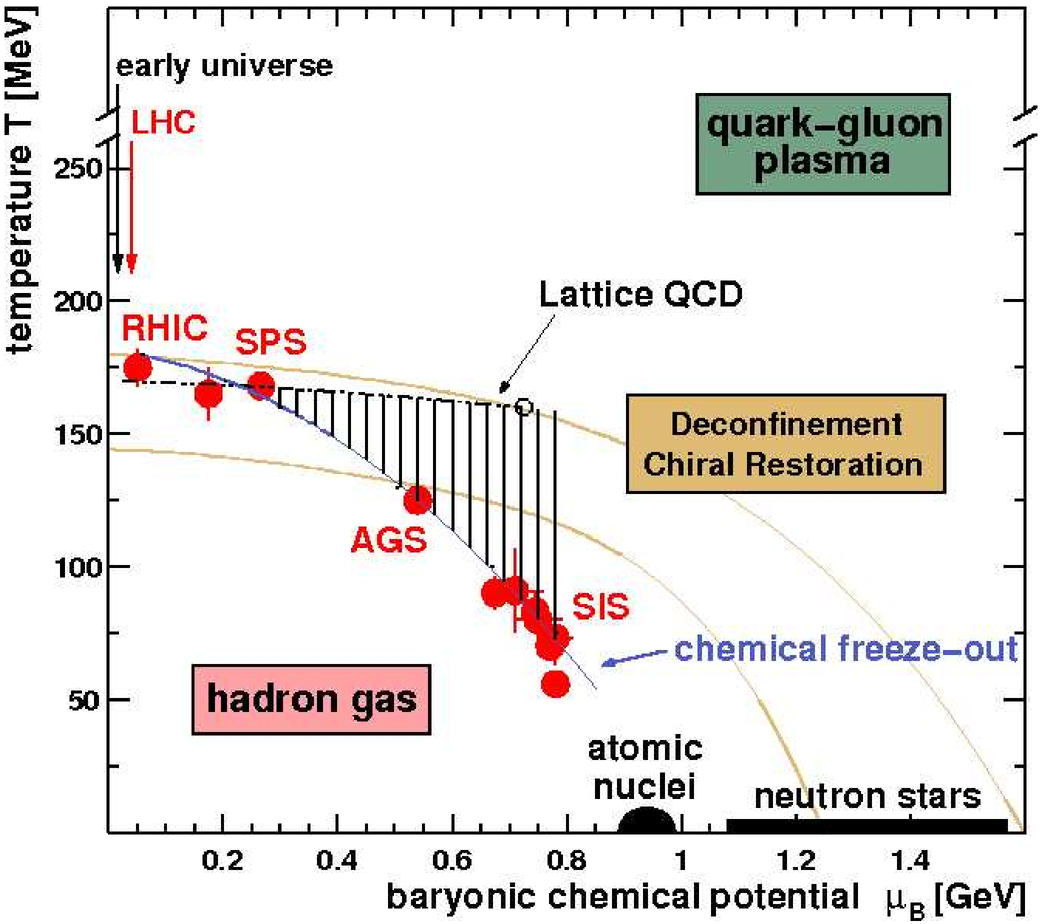
* This work was done with Prof G. Senjanović and Dr.B. Bajc, ICTP (Trieste, Italy)

Introduction

- In the early 1970's, the focus of much of the work in elementary particle physics was the unification of the weak and electromagnetic interactions.
- In the 1980's the focus shifted to the unification of strong,weak and electromagnetic interactions through spontaneously-broken grand unified theories (GUTs).
- The important feature of SSB in the masses of gauge bosons (and fermions) arises as a result of a non-zero vacuum expectation value of some scalar fields.
- Kirzhnits and Linde, *P.L.***B42***(1972)*, found that above some critical temperature the symmetry is restored and the VEV of scalar fields vanishes.
- Weinberg, *P.R.***D9** *(1974)*,and later on Mohapatra and Senjanović, *P.R.***D20***(1979)*, were the first to recognize the important applications of the symmetry nonrestoration at High temperature (the broken symmetries remain broken at arbitraly high temperatures).

Why symmetry at high temperature ?



Field theory and formalism - 1 -

- We start by defining nonzero temperature field theory: The ensemble of finite temperature Green's functions is defined by :

$$G_\beta(x_1, \ldots, x_n) = \frac{Tre^{-\beta H} T[\phi(x_1), \ldots, \phi(x_n)]}{Tre^{-\beta H}} \tag{1}$$

- Spontaneous symmetry violation is conveniently studied with the help of finite temperature effective action $\Gamma^\beta(\overline{\phi})$,

$$\Gamma^\beta(\overline{\phi}) = W^\beta(J) - \int d^4x \overline{\phi}(x) J(x) \tag{2}$$

where

$$\overline{\phi}(x) = \frac{\delta W^\beta(j)}{\delta J(x)}$$

$$W^\beta(J) = -i \ln Z^\beta(J)$$

Field theory and formalism - 2 -

$$Z^{\beta}(j) = \frac{Tre^{-\beta H}T[\exp(i\int d^4x\phi(x)J(x))]}{Tre^{-\beta H}}$$

- Symmetry violation is signaled by a nonvanishing value of $\overline{\phi}$, for which $\delta\Gamma^{\beta}(\overline{\phi})/\delta\overline{\phi}(x)$ is zero.
- The effective potential is then defined by :

$$V^{\beta}(\overline{\phi}) = -(space - time \quad volume)^{-1}\Gamma^{\beta}(\overline{\phi})\Big|_{\overline{\phi}=\phi} \quad (3)$$

and the symmetry breaking occurs when

$$\frac{\partial V^{\beta}(\phi)}{\partial \phi} = 0, \qquad \text{for} \quad \phi \neq 0 \quad (4)$$

Symmetry behaviour at finite temperature

- **Spontaneous Symmetry Breaking**(SSB) :
  We consider a theory with one self-interacting real scalar field

$$\mathcal{L} = \frac{\mathbf{1}}{\mathbf{2}}\partial_\mu\phi\partial^\mu\phi - \mathbf{V}(\phi) \tag{5}$$

  The classical field configuration with a minimum energy is given by a constant field $\phi(x) = \phi_0$, where ϕ_0 is taken to minimize the potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 \tag{6}$$

  Signs in $V(\phi)$ are opposite, it has two minima at $\phi_0 = \pm\upsilon = \pm\mu\sqrt{6/\lambda}$.

- The symmetry that is spontaneously broken or restored as $m^2 = 0$ in this very simple system is that under the exchange $\phi \to -\phi$ one vacuum (for example υ) changes to an other one $(-\upsilon)$, so clearly the vacuum is not invariant under this symmetry.

Symmetry behaviour at finite temperature

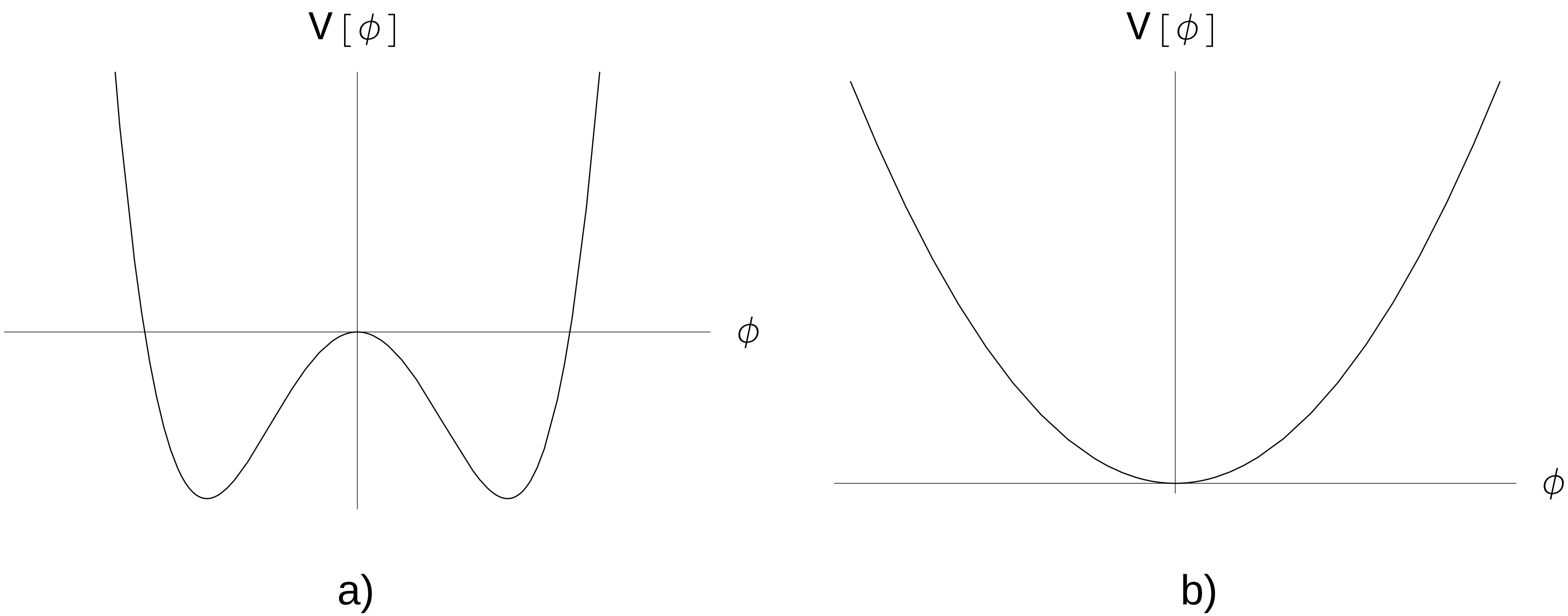


Figure 1: Potential $V(\phi)$ of real scalar field with ϕ^4-interaction corresponding to (a) situation with broken symmetry, and (b) situation with unbroken (restored) symmetry.

Symmetry behaviour at finite temperature

- The Aim is to express spontaneous symmetry breaking and restoration criteria in variational terms. The potential V satisfies the condition :

$$\left[\frac{\partial V}{\partial \phi}\right]_{\phi(x)=\phi_0} = 0 \qquad (7)$$

- The question is now: How do we decide, based on this variational condition on the potential, whether the system is in a spontaneously broken phase or in a fully symmetric one?

- It is clear that we have

$$\left[\frac{\partial V}{\partial \phi}\right]_{\phi(x)=\phi_0\neq 0} = 0 \Longleftrightarrow \text{Symmetry is spontaneously broken,} \qquad (8)$$

$$\left[\frac{\partial V}{\partial \phi}\right]_{\phi(x)=\phi_0=0} = 0 \Longleftrightarrow \text{Symmetry is unbroken.} \qquad (9)$$

Symmetry behaviour at finite temperature

- In order to reformulate the stationarity conditions (8)-(9) with the quantum theory in the thermal case ,

$$\left[\frac{\partial V^{\beta}_{eff}(\overline{\phi}_c)}{\partial\overline{\phi}_c}\right]_{\overline{\phi}_0=<\phi>_\beta\neq 0} = 0 \Longleftrightarrow \text{SSB by thermal effects,} \quad (10)$$

$$\left[\frac{\partial V^{\beta}_{eff}(\overline{\phi}_c)}{\partial\overline{\phi}_c}\right]_{\overline{\phi}_0=<\phi>_\beta=0} = 0 \Longleftrightarrow \text{Symmetry is restored by thermal effects.}$$

- Weinberg has written the finite-temperature effective potential as

$$V_{eff}(\phi,T) = V_{ren}(\phi) + \frac{1}{48}T^2\left[f_{ijkk} + 6(\theta_\alpha\theta_\alpha)_{ij} + Tr(\Gamma_i\gamma_4\Gamma_j\gamma_4)\right]\phi_i\phi_j \quad (11)$$

where $V_{ren}(\phi)$ is just the original $V(\phi)$ but with masses replaced by renormalized values.

Symmetry restoration and non-restoration

- The finite temperature contributions of scalar and gauge fields to the effective potential are :

$$\begin{aligned} \Delta V(T) &= \Delta V_{scalar}(T) + \Delta V(T)_{gauge}, \\ \Delta V_{scalar}(T) &= \frac{T^2}{24} \sum_{i=1}^{N} \frac{\partial^2 V}{\partial \phi_i^2}, \\ \Delta V_{gauge}(T) &= \frac{T^2}{24} 3g^2 \sum_{i,j=1}^{N} (T_a T_a)_{ij} \phi_i \phi_j. \end{aligned} \qquad (12)$$

where T_a are the group generators, and g is the gauge coupling constant. The finite temperature effective potential is

$$\overline{V} = V + \Delta V(T). \qquad (13)$$

Symmetry restoration and non-restoration

- The critical temperature is defined by:

$$T_C = \frac{\partial \overline{V}}{\partial \phi^2}\Big|_{\phi^2=0} \tag{14}$$

  If $T > T_C$ or $m^2(T) > 0$ the symmetry is restored, and the other way around if $T < T_C$ or $m^2(T) < 0$ the symmetry is broken.

- To see how the symmetry restores or does not restore at high temperature, we will consider two examples: a real scalar field theory with Z_2 symmetry and two real scalar field with symmetry $Z_2 \bigotimes Z_2$.

The Z_2 example of a symmetry restoration

- Let us begin by discussing an example above, i.e. a real scalar field theory which is described by a Lagrangian density

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4}(\phi^2 - \eta^2)^2 \tag{15}$$

- The boundedness of the potential in (15) requires $\lambda > 0$ and the minimum of this potential is $\phi^2 = \eta^2$ (in this theory the symmetry is spontaneously broken). By using (12), the one-loop temperature-dependent contribution to the effective potential is

$$\Delta V(T) = \frac{\lambda T^2 \phi^2}{8} \tag{16}$$

the temperature-dependent effective potential is (again only leading T^2 term)

$$\overline{V}(\phi, T) = \frac{\lambda}{8} T^2 \phi^2 + \frac{\lambda}{4}(\phi^2 - \eta^2)^2 \tag{17}$$

Z_2 example of a symmetry restoration

- The corresponding critical temperature in this case is

$$T_C = \left.\frac{\partial \overline{V}}{\partial \phi^2}\right|_{\phi^2=0} = 2\eta \tag{18}$$

From (13) and (14) we see that if

- $T < T_C$ the effective mass becomes negative and the symmetry is spontaneously broken
- However, when $T > T_C$ the effective mass term becomes positive and the symmetry restored. This leads to the restoration of the Z_2 symmetry.

$Z_2 \bigotimes Z_2$ example of a partial non-restoration

- It was noticed by Weinberg, and later on by Mohapatra and Senjanović, that the symmetry can be fully or partially restored depending on the range of parameters.
  Taking the simplest model, which consists of two real scalar fields and $Z_2 \bigotimes Z_2$ symmetry, with a zero temperature potential given by

$$V = \frac{\lambda_1}{4}\phi_1^4 + \frac{\lambda_2}{4}\phi_2^4 + \frac{\lambda}{2}\phi_1^2\phi_2^2 + \frac{\mu_1^2}{2}\phi_1^2 + \frac{\mu_2^2}{2}\phi_2^2 \qquad (19)$$

  The boundedness from below of this potential requires that $\lambda_{1,2} > 0$ and $\lambda_1\lambda_2 > \lambda^2$.

- At high temperature one uses the formula (12) to calculate the high temperature contribution to the potential (15) :

$$\Delta V(T) = \frac{T^2}{24}\sum_{i=1}^{N}\frac{\partial^2 V}{\partial \phi_i^2} = \frac{T^2}{24}(3\lambda_1 + \lambda)\phi_1^2 + \frac{T^2}{24}(3\lambda_2 + \lambda)\phi_2^2. \qquad (20)$$

$Z_2 \bigotimes Z_2$ example of a partial nonrestoration

- One can choose the parameters so that $(3\lambda_1 + \lambda)\phi_1^2 < 0$ and obtain a nonzero vev for the first field, $< \phi_1 > \neq 0$, spontaneously breaking in this way the first discrete symmetry Z_2.
    - Due to the boundedness conditions the same can not be done for the second Z_2.
    - In fact $(3\lambda_2 + \lambda)$ must now be positive, so the second field does develop a nonzero vev, i.e.$< \phi_2 >= 0$
- Since at very high temperature, the temperature itself is only the mass scale in the problem, the nonzero vev of ϕ_1 must be proportional to the temperature. So we have an example of symmetry nonrestoration, which persists at arbitrarly high temperatures.

Conclusion

The phenomenon of symmetry nonrestoration appears when the system has at high T less symmetry than allowed by the Lagrangian (some vev is nonzero) Commonly, people believe that the symmetry will be restored at high temperature.

However, as we have seen so far, this is not necessarily true. The symmetries may or may not get restored depending on the range of the parameters of the theory. We have given a simple example of the $Z_2 \bigotimes Z_2$ symmetries and showed that at high temperature the symmetry may get restored only partially.